$$S_0 = \sum_{k=0}^{n} q^k = \frac{q^{n+1}-1}{q-1};$$

$$S_1^* = \frac{\partial S_0}{\partial q} = \sum_{k=0}^{n} k\, q^{k-1} = \frac{(n+1)\, q^n (q-1) - (q^{n+1}-1)}{(q-1)^2} = \frac{q^{n+1}(n+1-1) - q^n(n+1)+1}{(q-1)^2}$$

$$= \frac{n\, q^{n+1} - q^n (n+1) + 1}{(q-1)^2};\ S_1 = \sum_{k=0}^{n} k\, q^k = \sum_{k=1}^{n} k\, q^k = q * S_1^*$$

$$\boxed{S_1 = \sum_{k=1}^{n} k\, q^k = \frac{n\, q^{n+2} - q^{n+1}(n+1) + q}{(q-1)^2}}$$

$$S_2^* = \frac{\partial S_1}{\partial q} = \sum_{k=1}^{n} k^2\, q^{k-1} = \frac{\big(n(n+2) q^{n+1} - (n+1)^2 q^n + 1\big)(q-1) - 2\big(n\, q^{n+2} - q^{n+1}(n+1) + q\big)}{(q-1)^3}$$

Zähler $= q^{n+2}\big(n(n+2) - 2n\big) + q^{n+1}\big(-n(n+2) + (n+1)^2 + 2(n+1)\big) + q^n (n+1)^2 - q - 1$

Zähler $= q^{n+2}\, n^2 + q^{n+1}\,(2\,n+1) + q^n\,(n+1)^2 - q - 1$

$$\boxed{S_2 = q\, S_2^* = \frac{q}{(q-1)^3}\big(q^{n+2} n^2 + q^{n+1}(2n+1) + q^n (n+1)^2 - q - 1\big)}$$

Für $n \to \infty$ bleibt also $\dfrac{q(1-q)}{(1-q)^3}$